STAC63 Midterm Test

Due by: Thursday, February 25 at 13:00 EST via Crowdmark
Number of questions: 4 Time: 2 hours + 20 minutes Total points available: 40

- 1. (10 points) This question has two independent parts (a) and (b).
 - (a) Let X_1 and X_2 be independent geometric random variables having the same parameter p. Guess the value of

$$\mathbb{P}(X_1 = i|X_1 + X_2 = n).$$

(b) An unbiased die is successively rolled. Let X and Y denote, respectively, the number of rolls necessary to obtain a six and a five. Find $\mathbb{E}[X]$, $\mathbb{E}[X|Y=1]$ and $\mathbb{E}[X|Y=5]$.

2. (10 points) Consider a Markov chain with state space $S=\{1,2,3,4\}$ and transition probabilities

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

- (a) Compute f_{43} and f_{23} .
- (b) Compute $\lim_{n\to\infty}\sum_{n=1}^{\infty}p_{22}^{(n)}$ and determine if state 2 is recurrent or transient.

3. (10 points) Consider a Markov chain with state space $S=\{1,2,3\}$ and transition probabilities

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0\\ 0 & \frac{1}{3} & \frac{2}{3}\\ \frac{3}{4} & 0 & \frac{1}{4} \end{pmatrix}.$$

- (a) Show that the Markov chain is irreducible and aperiodic.
- (b) Compute $\lim_{n\to\infty} p_{12}^{(n)}$.

- 4. (10 points) Suppose cars can be blue, green or red and that a car will be the same colour with probability 50% and equally likely to be a different colour with probability 25%. Moreover, suppose you know the last car that passed by was blue. For example, if the last car was blue, then there is a 50%, 25% and 25% chance that the next car will be blue, green or red respectively.
 - (a) Write out the transition probability matrix.
 - (b) Calculate the expected number of cars until the pattern red-green (a green car passes after a red one) first occurs.