Final Exam MATA33 (Summer 2019, UTSC)

You have 3 hours for this final exam. There are 100 points in total. No aids allowed. Any violation of academic integrity is forbidden and will be prosecuted. You need to get 40 points on this final exam to pass the course MATA33. Your solutions must be written down in a clear, consistent and explicitly complete way. You are required to clearly fill in your name and student number below (otherwise your exam is not accepted).

Total Points Calculation

Problem 1 :	/6	Problem 7 :	/6
Problem 2 :	/6	Problem 8 :	/6
Problem 3 :	/8	Problem 9 :	/12
Problem 4 :	/6	Problem 10 :	/14
Problem 5 :	/6	Problem 11 :	/12
Problem 6 :	/6	Problem 12 :	/12
		Total :	/100

Part I: Short Answer (50pts)

Problem 1: Determinants

(6pts) Let A, B and C be 2×2 square matrices with $\det(A) = 1$, $B = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$.

(1a) **(2pts)** Find $\det(2AB^2)$.

(1b) **(2pts)** Find $\det\left((\frac{1}{2}B)^{-1}C\right)$.

(1c) **(2pts)** Find $\det(AB + AC)$.

Continue your solution of Problem 1 here.

Problem 2: System of Equations

(6pts) Solve the following linear system using row reduction.

$$\begin{cases} 2x + z & = 21 \\ 3x + 2y - 3z & = 34 \\ -x - 3y + 5z & = -18 \end{cases}$$

Continue your solution of Problem 2 here.

Problem 3: Matrix Inverse

(8pts) Let $A = \begin{pmatrix} 2 & 0 & -2 \\ 3 & 1 & 2 \\ 1 & 0 & -3 \end{pmatrix}$.

(3a) **(2pts)** Determine if A is invertible.

(3b) **(6pts)** Using the method of cofactors, find A^{-1}.

Continue your solution of Problem 3 here.

Problem 4: Chain Rule

(6pts) Let $z(x, y) = 2x^2 \ln|3x - 5y|$ where $x(s, t) = s\sqrt{t^2 + 2}$ and $y(s, t) = t - 3e^{2-s}$. Evaluate z_t at $(s, t) = (1, 0)$.

Continue your solution of Problem 4 here.

Problem 5: Implicit Differentiation

(6pts) Let $xz^2(x,y) - y^2z(x,y) = 1$. Find z_{xx}.

Problem 6: Critical Points

(6pts) Find and classify all the critical points of

$$f(x, y) = \ln(xy) + 2x^2 - xy - 6x.$$

Problem 7: Lagrange Multipliers

(6pts) Find the extrema of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $z = x^2 + y^2$ and $x + y + z = 12$.

Problem 8: Integration

(6pts) Let $f(x, y) = e^{-2x-3y}$ for $x, y \geq 0$. Evaluate

$$\int_1^3 \int_2^4 f(x, y)\, dy\, dx.$$

Part II: Long Answer (50pts)

Problem 9: Stöckli Skis

(12pts) Stöckli Skis is a world-class manufacturer of two types of race stock skis: the Laser SL FIS and Laser GS FIS. Pairs of slalom (SL) and giant slalom (GS) skis gross \$1000 and \$1500 in revenue and realize net income margins of 25% and 20% respectively. The factory can produce at most $60,000$ skis per annum. The management team is willing to reinvest its \$54MM in retained earnings from FY19. How many slalom and giant slalom skis should Stöckli Skis manufacture to maximize net income for FY20? How much income is realized?

Continue your solution of Problem 9 here.

Problem 10: Structural Composite

(14pts) A mature construction materials start-up has developed a structural composite material with factories in three regional markets: AMER (Americas), EMEA (Europe, Middle East and Africa) and APAC (Asia Pacific). The composite realizes \$400, \$100 and \$100 in revenue per labour hour in each of the three regions respectively and is driven by a production function

$$Q(x, y, z) = x^{2/5} y^{2/5} z^{1/5}.$$

(10a) **(10pts)** The start-up has a production quota of $10,000$ units. How should the start-up allocate its production quota to maximize gross revenue? How much revenue is realized?

(10b) **(4pts)** The marginal cost of production is \$800. What is the marginal revenue (of) product? Is the start-up profitable?

Continue your solution of Problem 10 here.

Problem 11: Kicking Horse

(12pts) A skier finds herself at Kicking Horse and wants to divide her time between skiing powder, groomers and bumps. She will spend 5%, 10% and 5% of her time waiting for lifts and another 20%, 15% and 10% of her time riding lifts to ski powder, groomers and bumps respectively. She plans to ski for a total of 6 hours, spending 30 minutes waiting for lifts and 1 hour riding lifts. How does she allocate her time? How much time is she actually skiing?

Continue your solution of Problem 11 here.

Problem 12: Lake Louise

(12pts) A ski racer and his buddy find themselves at the top of Men's Downhill on Whitehorn Mountain at Lake Louise. Their coordinates are both $(x, y) = (1, 1)$ and the mountain's height is

$$z(x, y) = \frac{1}{10} e^{7-4x^2-3y^2}.$$

(12a) **(8pts)** The more audacious ski racer would like to ski downhill along the steepest possible path. What is his direction of steepest descent? What is the slope in that direction?

(12b) **(4pts)** The racer's less intrepid buddy would also like to ski downhill, but only along paths that are at most 50% as steep as the path of the racer. What is the minimum angle he could traverse with respect to the racer? The maximum?

Continue your solution of Problem 12 here.

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